SINGLE AND MULTICHANNEL SAMPLING OF BILEVEL POLYGONS USING EXPONENTIAL SPLINES

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ABSTRACT

The contribution of our work is two-fold; First, we present an algorithm for sampling and perfectly reconstructing bilevel polygons using exponential spline (E-spline) sampling kernels [1] by using the fact that, with E-splines as sampling kernels, we can obtain the Fourier coefficients of $g(x, y)$ from the samples. Then we consider the case where a bank of E-spline filters is used to acquire a polygonal image as the input signal, where each filter has access to a delayed version of the input signal. It will be shown that, by registering the delay parameters from the relevant features of the image samples, it is possible to synchronize the different channels exactly so that perfect reconstruction of the original polygonal image and its delayed versions is achieved at the receiver.

For the single sampling case, S.Lee and R.Mittra [2] derived a general formula for the Fourier transform of any K-sided bilevel polygon where they showed that the Fourier transform is directly related to the location of the polygon's vertices. With the use of Radon transform and its direct relationship with the Fourier transform we can transform the Fourier coefficients of bilevel polygons, obtained from E-spline sampling kernel, to the Radon domain. The new mapped equation, at different projections, follows the data model used for the 1-D harmonic retrieval data model exactly, thus by using the Prony's method, we can find all the necessary parameters. Any K-sided convex and bilevel polygon is completely specified by the location of its K vertices, therefore, as Maravic points out in her paper [3], K+1 projections will guarantee us to perfectly retrieve the vertices of the bilevel polygon. By projections we mean line integrals at arbitrary angles $\tan^{-1}(\frac{n}{m})$, where m and n are the indices of the samples. It is shown that the minimum spline order required for a perfect reconstruction of a given K-sided bilevel polygon is $N = p.(2K - 2)$ where $p = max(m, n)$ needed in order to produce at least K+1 projections.

We also investigate the scenario of multichannel sampling of bilevel polygons where a bank of E-spline filters receive different delayed versions of the original image $g_0(x, y)$. Baboulaz [4] has looked at the case of multichannel sampling of a stream 1-D Diracs using exponential splines, with simple translation being the delay or the transformation parameter, which he proves that, unlike the polynomial reproducing kernels, we can truly distribute the acquisition of FRI signals with kernels reproducing exponentials. The reason is that, exponential splines can offer different kernels with the same order. The method discussed in [4] method can be extended to the bilevel polygons' case where we only have translations x_0 and y_0 as the transformation parameters. In this scenario, not only we can exactly retrieve the shifts x_0 and y_0 but we also can easily produce the rest of the samples of both images all together. This means that, by estimating the shifts, we can produce the higher order moments of the reference image from the lower order moments of its translated version and vice versa. Finally we can run our algorithm described in the single sampling case to perfectly reconstruct the bilevel polygon. Multichannel sampling aims to have sensors of lower order. Since we need less samples from each sensor, the support of the corresponding sampling kernels are also reduced resulting in a symmetrical, synchronized and a more stable system. It is important to mention that generally, using a multichannel system to acquire a scene makes the system more robust to noise and sensor failure.

1. REFERENCES

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